Cryptarithmetic problems with explanations

Cryptarithmetic problems are where numbers are replaced with alphabets. By using standard arithmetic rules we need to decipher the alphabet.

General Rules:

1. Each alphabet takes only one number from 0 to 9 uniquely.
2. Two single digit numbers sum can be maximum 19 with carryover. So carry over in problems of two number addition is always 1.
3. Try to solve left most digit in the given problem.
4. If a × b = kb, then the following are the possibilities

(3 × 5 = 15; 7 × 5 = 35; 9 × 5 = 45) or (2 × 6 = 12; 4 × 6 = 24; 8 × 6 = 48)

Solved Example 1:

The following questions are based on the following multiplication, where each digit has been replaced by an alphabet.

		J	E
	x	B	B
		J	E
	J	E	A
B	**A**	**D**	**E**

1. Find the value of J and A.
2. If E = 4, what is the value of D?

Explanation:

From the first two of multiplication, you can clearly say that B = 1, as JE x B = JE. From the second row of multiplication, A = 0 as in the multiplication, second row should start from tenth's place. So A = 0. Now in the hundred's place, J + Something = 10. When you add something to the single digit number that results in 10. So J = 9. Now from the above table, we cannot determine values of E and D, but we can say, that E and D are consecutive. As it is given that E = 4, we can say D = 3.

		9	E
		1	1
		9	E
	9	E	0
1	**0**	**3**	**E**

Solved Example 2:

From the multiplication below, What is the value of NAME?

		H	E
	x	E	H
		H	E
H	H	A	
H	**N**	**M**	**E**

Explanation:

From the first row of multiplication, H =1 is clear, As HE x H = HE. Substitute H = 1 in all places. Now from the tenth's place, think about, the value of A. 1 + A = M. If M is a single digit number, then N = 1, which is impossible (Already we have given H = 1). So A = 9, Then M = 0, and N = 2. Now 1E x E = 119. So by trial and error E = 7.

Therefore, NAME = 2907

		1	7
		7	1
		1	7
1	1	9	
1	**2**	**0**	**7**

Solved Example 3:

Decipher the following multiplication table

			E	Y	E
			M	A	T
		S	Y	I	A
	G	M	T	A	
A	I	R	Y		
A	**A**	**S**	**M**	**A**	**A**

Explanation:

Step 1: What could be the value of A which is the left most digit in the answer? From the second row of multiplication, we know that A x E = A. So A cannot be 1.

From the tenth's place addition, I + A = A. So I = 0. Now from the Ten-thousand's place addition, 1 + G + 0 = A. So G = 1 and A = 2.

			E	Y	E
			M	2	T
		S	Y	I	2
	1	M	T	2	
2	0	R	Y		
2	**2**	**S**	**M**	**2**	**2**

Step 2: From the second row of multiplication, 2 x E = 2, Also from first row of multiplication T x E = 2.

			E	Y	E
			M	2	T
		S	Y	0	2
	1	M	T	2	
2	0	R	Y		
2	**2**	**S**	**M**	**2**	**2**

So E should take 6, and T should take 7. (These are the only possibilities)

If E= 6, then E x T = 6 x 7 = 42. So 4 carry over. Now 7 x Y + 4 = 0. So Y = 8.

			6	8	6
			M	2	7
		S	8	0	2
	1	M	7	2	
2	0	R	8		
2	**2**	**S**	**M**	**2**	**2**

Now S = 4 as 7 x 6 + 6 = 48. Also M = 3.

Final answer looks like this:

			6	8	6
			3	2	7
		4	8	0	2
	1	3	7	2	
2	0	5	8		
2	**2**	**4**	**3**	**2**	**2**

Solved Example 4:

If SEND + MORE = MONEY then find the respective values

Explanation:

Addition of two numbers with 'n' digits, results in a n+1 digits, then the left most place always = 1.

So M=1. Substitute this value.

Now 'o' cannot be 1 as M already 1. It may not be 2 either as S+1 = 12 or 1 + S + 1 = 12 in the both cases S is a two digit number. So 'o' is nothing but zero. Put o = 0.

Now S can be either 8 or 9. If S = 8, then there must be a carry over.

E + 0 = 10 + N or 1 + E + 0 = 10 + N

In the above two cases, E - N = 10 is not possible and E - N = 9 not possible as as N cannot be zero.

So E = 9.

Now E + 0 = N is not possible as E = N. So 1 + E = N possible.

		1		
	9	E	N	D
	1	0	R	E
1	**0**	**N**	**E**	**Y**

The possible cases are, N + R = 10 + E - - - (1) or 1 + N + R = 10 + E - - - (2)

Substituting E = N -1 in the first equation, N + R = 10 + N - 1, we get R = 9 which is not possible.

Substituting E = N - 1 in the second equation, 1 + N + R = 10 + N - 1, we get R = 8.

We know that N and E are consecutive and N is larger. Take (N, E) = (7, 6) check and substitute, you wont get any unique value for D.

Take (N, E) = (6, 5), Now you get D = 7, Y = 2.

	9	5	6	7
	1	0	8	5
1	**0**	**6**	**5**	**2**

Solved Example 5:

Find the values of all the alphabets if each alphabet represent a single digit from 0 - 9

	A	B	C	D
+	E	B	C	B
A	F	G	A	G

Explanation:

Let us name the columns as below

	1	2	3	4
	A	B	C	D
+	E	B	C	B
A	F	G	A	G

We know that sum of two single digit alphabets should not cross 18, and maximum difference between two alphabets is 9.

If we add two maximum 4 digit numbers the sum is maximum 19998. So the digit in the 5th left is 1.

Now from the 1st column 1 + E = 1F; if there is any carry over from the 2nd column 1 + 1 + E = 1F

But 1F is a two digit number in alphanumeric is equal to 10 + F

So $1+E=10+F \Rightarrow E-F=9$

From this relatlion we know that E = 9, F = 0

or $1+1+E=10+F \Rightarrow E-F=8$

E = 9, F = 1 or E = 8, F = 0

From the above we can infer that F = 0 but we dont know whether E is equal to either 8 or 9. But surely F is not equal to 1 as we fixed already A = 1

Now from the 3rd column

2C= 1 ⇒ C = 1/2

1 + 2C = 1 ⇒ C = 0

If the sum is a two digit number then

2C = 11 ⇒ C= 11/2

1 + 2C = 11 ⇒ C = 5

From the above C = 1/2 and 11/2 are not possible nor is 0 possible as we fixed F = 0

If C = 5 the the A = 1 and there is a carry over to the left column. and also there must be carry over from the first column, but we dont know 1 + 2B is a single digit or two digit number

From the second and fourth columns

1+2B = G - - - - (1) or 1 + 2B = 10 + G - - - (2)

D + B = 10 + G - - - (3)

Solving (1) and (3) we get D - B = 11 which is not possible

But If we solve (2) and (3) then we get D - B = 1

So D and B are consecutive numbers and their sum is more than 10. So acceptable values are D = 7 and B = 6

This completes our problem so final table looks like the following

	A	B	C	D			1	6	5	7
+	E	B	C	B		+	8	6	5	6
A	F	G	A	G		1	0	3	1	3

Solved Example 6:

Find the alphabets in the following multiplication

			P	A	S
			R	B	Q
		S	B	K	W
	A	S	A	A	
S	E	P	B		
S	**Q**	**S**	**K**	**A**	**W**

Explanation:

This is a tough question as there are total 9 different alphabets are used.

Step 1: K + A = A. So K = 0

Step 2: From the hundreds column, 2B + A = 10 or 20. As 2B, 10, 20 are even, A should be even. Remember this logic.

Possibilities are, for A and B are (2, 4), (4, 3), (6, 2), (8, 1) and (2, 9), (4, 8), (6, 7), (8, 6)

In the second row of multiplication, we have PAS x B = ASAA.

P2S x 4 = 2S22 ⇒S = 3, 8 But both are not satisfying.

P4S x 3 = 4S44 ⇒S = 8. But P48 x 3 = 4844 is not possible. Ruled out.

P6S x 2 = 6S66 ⇒S = 3, 8. But both are not satisfying. Ruled out.

P2S x 9 = 2S22 ⇒S = 8 But P28 x 9 = 2822 is not possible. Ruled out.

P4S x 8 = 4S44 ⇒S = 3. This is possible as P43 x 8 = 4344 then P = 5.

P6S x 7 = 6S66 ⇒S = 8 But P68 x 7 = 6866 is not possible. Ruled out.

P8S x 6 = 8S88 ⇒S = 3, 8 But both are not satisfying. Ruled out.

Therefore, S = 3, P = 5, A = 4, B = 8.

			5	4	3
			R	8	Q
		3	8	0	W
	4	3	4	4	
S	E	5	8		
3	**Q**	**3**	**0**	**4**	**W**

From the above diagram, R = 6 and E = 2. and A = 7 and W = 1.

Final Solution:

			5	4	3
			6	8	7
		3	8	0	1
	4	3	4	4	
S	2	5	8		
3	**7**	**3**	**0**	**4**	**1**

Solved Example 7:

Find the values of A, B and C if $ABC = A! + B! + C!$ where ABC is a three digit number

Explanation:

By symmetry, we take A is the maximum number of the three alphabets.

Let us say A can take a maximum value of 6.

Then 6! = 720 but as have we taken maximum value is 6 720 is not possible

So A should take 5. Then 5! = 120

Now from the above we know that one the B and C should take 1 as their value as 120 consists of 1.

⇒ 5! + 1! = 121

If we take 4 as one of the number then 5!+ 1! + 4! = 145 or 1! + 4! + 5! = 145

Solved Example 8:

Find the values of A, B and C if $ABC = A^3 + B^3 + C^3$ where ABC is a three digit number.

Explanation:

Let us say maximum value of A, B, C is equal to 4 then we dont get any satisfactory values for ABC

If we take maximum value is 5 then A^3 = 5^3 = 125 as this is a three digit number one of the number is equal to 1.

$\Rightarrow$ $5^3 + 1^3$ = 126. Now for 3^3 we get $153 = 1^3 + 5^3 + 3^3$

From the above reasoning the other numbers satisfy the above relation are 370, 371 and 407